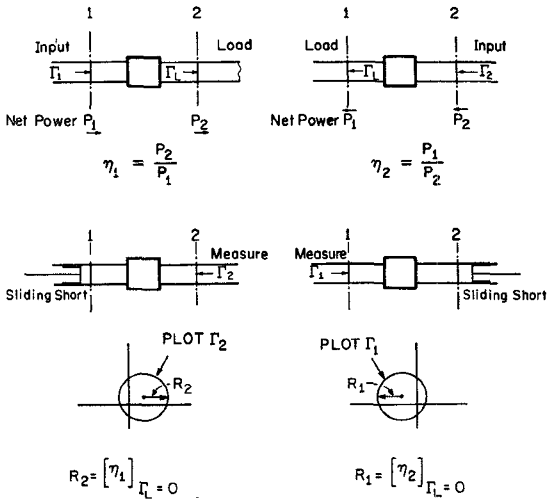

Fig. 1. Diagrams illustrating graphical method of obtaining 2-port efficiencies from reflection coefficient measurements.

and measuring the radii of their circular loci. These radii can be shown to be closely related to the desired efficiencies.

Procedure

Consider that Γ_1 has been measured corresponding to various positions of the sliding short circuit in arm 2 of a 2-arm waveguide junction. A circle has been drawn through the measured points. The radius of the Γ_1 circle equals η_{2m}. Now suppose that we wish to determine η_2 for a different termination on arm 1 having a reflection coefficient Γ_L. We use the data for Γ_1 and calculate new reflection coefficients Γ_{1N} by the formula

$$\Gamma_{1N} = \frac{\Gamma_1 - \Gamma_L}{1 - \Gamma_1 \Gamma_L}. \tag{2}$$

Since Γ_{1N} is a linear fractional transformation of Γ_1, it has a circular locus of radius R_{1N}. One plots Γ_{1N}, measures R_{1N}, then calculates

$$\eta_2 = \frac{R_{1N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}} \tag{3}$$

where ψ_L is the phase of Γ_L.

In a similar way one may obtain from data on Γ_2,

$$\eta_1 = \frac{R_{2N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}}. \tag{4}$$

Theory

First we derive an expression for the radius of the circular locus of R_{1N}. We note that

$$\Gamma_1 = \frac{(S_{12}S_{21} - S_{11}S_{22})e^{i\phi} + S_{11}}{1 - S_{22}e^{i\phi}} \tag{5}$$

where ϕ is the phase of the sliding short-circuit termination. Consider the following equation

$$\Gamma_{1N} = \frac{(S_{12}S_{21} - S_{11}S_{22})e^{i\phi} + S_{11} - \Gamma_L(1 - S_{22}e^{i\phi})}{1 - S_{22}{}^{i\phi} - [(S_{12}S_{21} - S_{11}S_{22})e^{i\phi} + S_{11}]\Gamma_L}. \tag{6}$$

This can be cast in the form

$$\Gamma_{1N} = \frac{ae^{i\phi} + b}{ce^{i\phi} + d} \tag{7}$$

and the radius of the Γ_{1N} circle is [1]

$$R_{1N} = \frac{|ad - bc|}{|d|^2 - |c|^2} = \frac{|S_{12}S_{21}(1 - \Gamma_L{}^2)|}{|1 - S_{11}\Gamma_L|^2 - |(S_{12}S_{21} - S_{11}S_{22})\Gamma_L + S_{22}|^2}. \tag{8}$$

If we apply the reciprocity condition $Z_{01}S_{21} = Z_{02}S_{12}$

$$R_{1N} = \frac{Z_{02}}{Z_{01}} \frac{|S_{12}|^2 |1 - \Gamma_L{}^2|}{|1 - S_{11}\Gamma_L|^2 - |(S_{12}S_{21} - S_{11}S_{22})\Gamma_L + S_{22}|^2}. \tag{9}$$

However, the efficiency η_2 has been shown [1] to be equal to the right side of (9) multiplied by $1 - |\Gamma_L|^2 / |1 - \Gamma_L{}^2|$.

Thus

$$\eta_2 = R_{1N} \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L{}^2|} = \frac{R_{1N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}}. \tag{3}$$

It can be shown in a similar way that if

$$\Gamma_{2N} = \frac{\Gamma_2 - \Gamma_L}{1 - \Gamma_2 \Gamma_L} \tag{10}$$

and R_{2N} is the radius of the Γ_{2N} circle, then

$$\eta_1 = \frac{R_{2N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}}. \tag{4}$$

Conclusion

We can obtain the efficiency for any termination from the data used to plot the Γ_1 circle.

The additional labor of transforming the measured reflection coefficient to a new reflection coefficient according to (2) can be easily and quickly done on a desk-top programmable electronic calculator or a time-share computer terminal. Thus the method is a simple extension of previous methods involving a small amount of additional calculation but no additional taking of data. Compared to the method described by Mathis [4], it requires slightly less data and is potentially more accurate.[1]

It is of course possible to obtain η_1 and η_2 from only three measured reflection coefficients (either Γ_1 or Γ_2) if the corresponding terminations are known. However, the accuracy of the above method using circular loci can be much better because such curve fitting tends to "average out" errors in the individual measurements.

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[1] It has been shown that from the same data, plus one additional measured point with the 2-port terminated in the arbitrarily selected load, one can obtain the efficiency corresponding to that termination. This is done by means of a graphical construction and additional calculation. See [4].

Focusing of 52-GHz Beams by Cylindrical Mirrors

J. A. ARNAUD, D. C. HOGG, and J. T. RUSCIO

The possibility of using inexpensive commercial glass mirrors for refocusing and redirecting millimeter-wave beams [1] has been investigated; such beams would be useful for distributing large quantities of information in cities [2], [3]. Interference is expected to be minimal as a result of the close confinement of the beams. We report preliminary experiments made in the 50–55-GHz band, using a swept backward-wave oscillator (BWO) as a source.

A closed triangular path (25 m+35 m+25 m) incorporating a beam launcher, two refocuser/redirectors, and a beam collector has been set up on Crawford Hill, Holmdel, N. J. A refocuser is shown in Fig. 1.

The beam launcher and the beam collector are of the periscopic

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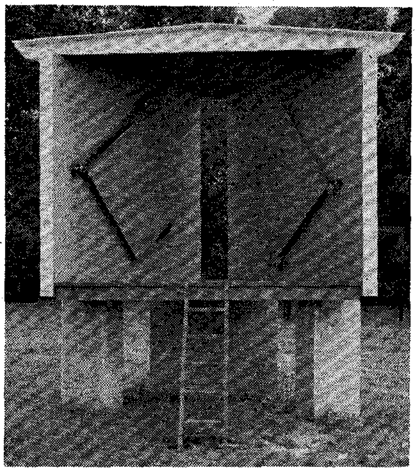

Fig. 1. One of the two refocusers used in the experiment showing the two cylindrical mirrors attached on wood panels. The generatrices of these mirrors make a 50.5° angle with the vertical direction. A 135° beam axis deflection is provided by the system.

type [4] and make use of 30-in (0.75-m) parabolic dishes and dual-mode feeds. The field radiated by these antennas was found to have an almost Gaussian amplitude distribution with a radius ($1/e$ point) equal to 0.2 m. The convergence of the launched beam can be varied by displacing the feed along the paraboloid axis.

Each refocuser incorporates two 4-ft by 4-ft (1.2-m by 1.2-m) cylindrical mirrors whose angular orientation is chosen such that the same focusing effect is produced in every meridional plane, as discussed in [5]. These mirrors are commercial 0.25-in (6-mm) thick glass mirrors bent to the required radius of curvature by exerting bending moments at the edges. They are used as front-surface mirrors, the coating protecting the copper plating being transparent to millimeter waves. Fine-thread screws allow the mirrors to be tilted in two directions for purposes of alignment.

Fig. 2 shows the measured 52-GHz round-trip loss as a function of the deformation at the center of the mirrors (the same for all four mirrors). For these data, the transmitted power is referred to the power measured when the launcher and collector feeds are directly coupled to one another. The measured attenuation, therefore, includes the losses of the launching and collecting antennas (resulting from blocking, spillover, ohmic losses, and wavefront distortion). The round-trip attenuation, which is as high as 7 dB when the mirrors of the refocusers are flat, drops to 1.5 dB when they are curved with optimum radii. This 1.5-dB attenuation is to be compared with the 1.3-dB loss observed when the launcher and collector face each other with a separation of about 2 m. These results indicate that the loss per refocuser is as low as 0.1 ± 0.05 dB. It should be noted, however, that this evaluation is based on the assumption that the losses add in decibels, an assumption that may not be completely justified if part of the loss results from wavefront distortion or spillover.

The curve in Fig. 2 gives the theoretical [6] minimum loss resulting from mismatch between the launched beam and the beam accepted by the collector under the assumption that the beams are Gaussian in shape and that spillover at the refocusers can be neglected. The measured launcher and collector losses (1.3 dB) and the absorption by the oxygen line of the atmosphere (0.085 dB) have been added to the calculated loss. There is rather close agreement between this curve and the experimental points.

When the experimental results are extrapolated to higher frequencies, such as 100 GHz, one finds that the total loss in the system during clear atmospheric conditions would not exceed 10 dB for a 1-mi long link. Note that, at 100 GHz, with the same mirror size, the separation between adjacent refocusers can be extended to 100 m. A 1-mi link would therefore incorporate 15 refocusers. Recent measurements [7], [8] indicate that the attenuation by rainstorms in New Jersey exceeds 40 dB/mi only 0.01 percent of the time at 100 GHz. A total loss of $40+10=50$ dB appears quite tolerable, even for large capacity systems, if 50-mW IMPATT oscillators are used as sources. An extension of the reported experiment to higher frequencies and longer path lengths is planned for the near future.

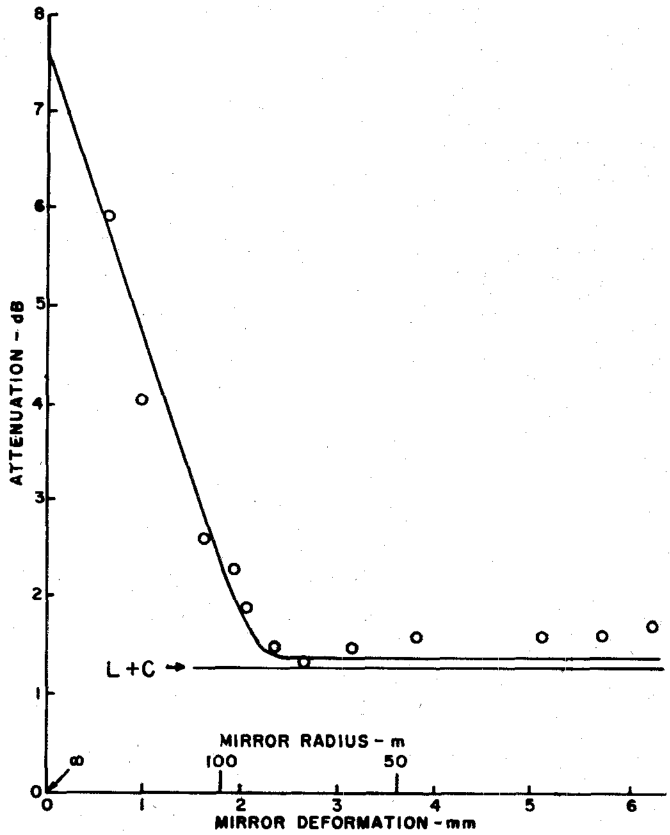

Fig. 2. The circles in this figure are the measured round-trip attenuations around the path as a function of the radius and maximum deformation of the four mirrors. The straight line ($L+C$) is the loss observed when the launcher and collector antennas face each other. The theoretical attenuation is represented by the curve.

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Correction to "A 10-GHz Single Sideband Modulator with 1-kHz Frequency Shift"

J. D. E. BEYNON

In the above correspondence[1] the fourth sentence of the fourth paragraph should read as follows: "For an error of less then 2 percent (not 1 percent) in the measurement of attenuation and 1° in that of phase shift, the unwanted sideband must be suppressed by at least 40 dB (assuming an infinite carrier suppression)." We apologize for this mistake.

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